Covariance Matrix Reconstruction Method for Mainlobe Interference Cancelling

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Abstract: As we know, the mainlobe interference always cause the adaptive beam pattern distortion which leads to the performance degradation. To over the problem, in this paper, a novel method is proposed to remove the mainlobe interference from the covariance matrix based on interference-plus-noise covariance matrix reconstruction. In which the direction of each interference is estimated to form separated angular sectors which are regarded as integral intervals. Then we use the Capon spectrum as an estimate of the spatial power spectrum to integral over the angular sectors to get the reconstructed covariance matrix. Furthermore the eigen-projection matrix preprocessing method and the block matrix preprocessing method is compared, while the EMP method is improved by using the eigenvector corresponding to the maximum eigenvalue of the reconstructed covariance of the mainlobe interference. Finally, the adaptive weight is obtained based on the reconstructed covariance matrix for the proposed method. Theoretical analysis and simulation results show the effectiveness and robustness of the proposed method.

1. Introduction

Adaptive beamformers have an excellent performance in suppressing sidelobe interferences, and is widely used in many fields, such as seismology radar, sonar, navigation, wireless communication, radio astronomy medical imaging, and so on[1-4]. As the data-dependent beamfomers, the adaptive weight vector is adjusted according to the signal environment to form an ideal mainlobe in the desired direction and to form nulls in the directions of interferences to suppress interferences and noise[5, 6], whose principle is to utilize the array gain to suppress the interferences, such as the well-known Capon beamformer[7, 8]. As we know, adaptive beamformers can form nulls in the direction of the side lobe interference which would be the same when the interference falls into the mainlobe[9]. In this scenario, the mainlobe experiences distortion, causing the maximum gain direction to deviate from the desired direction. As a result, the overall gain of the array suffers degradation.

To solve this problem, some algorithms have been developed in recent years, such as the block matrix preprocessing(BMP) method [10] which utilizes the prior direction information of the mainlobe interference to construct a block matrix which is used to process the received data to eliminate the mainlobe interference. The eigen-projection matrix preprocessing(EMP) method [11] which processes the received data with a eigen-projection matrix constructed by the eigenvector of the mainlobe interference. And the improved method [12] based on eigen-projection processing and covariance matrix reconstruction(EPCM) provides a simple way to get the eigenvector. The method based on polarization sensitive array [13] which deals with the mainlobe interference from the view of polarization can achieve a good performance by using the difference polarization information. However these methods have its own limitations for the BMP method is sensitive to the estimated direction angle of the mainlobe interference which means a small error will lead to performance degradation and on the other hand, degrees of freedom will decrease as the interference is cancelled. For EMP, as analyzed in [12], it is difficult to find the eigenvector of the mainlobe interference. Besides, the mainlobe peak offset is usually occurred in BMP and EMP method. For EPCM, nulls formed in the directions of the sideloe interference are not very deep and the output SINR still suffer a loss comparing to the optimal SINR[14].

To further explore the mainlobe interference suppression method, in this letter, a mainlobe interference suppression method is proposed based on the interference-plus-noise(IPN) covariance matrix reconstruction[15, 16]. In the proposed method, directions of interferences are firstly estimated by using some direction finding methods. To collect more potential information, the Capon spectrum [17] is act as an estimation of the spatial power spectrum and is used to integral over separate angular sectors according to each estimated direction to form new covariance matrices of each interference. The interference-plus-noise covariance matrix can be reconstructed by adding up the reconstructed covariance matrixes except the mainlobe one. Finally By using the reconstructed IPN, we can get the adaptive weight vector.

2. Problem Background

2.1 Signal Model

Consider an array composed of M sensors that receives and D+1 uncorrelated interferences including a mainlobe interference [18]. Let $x(k)$ denote the $M\times 1$ complex vector at the kth snapshot

$$\boldsymbol{x}(k)=\boldsymbol{a}(\theta_1)s_1(k)+\sum_{i=2}^{D}\boldsymbol{a}(\theta_i)s_i(k)+\boldsymbol{n}(k) \quad (1)$$

where (θ_i) $i=1,2,\cdots,D$ is the SV of mainlobe interference and sidelobe interferences. θ_i is the according interference direction. $s_i(k)$ is the complex envelope. $n(k)$ is the additive noise vector which is spatially temporally white Gaussian with equal variance.

The output can be represented as

$$\boldsymbol{y}=\boldsymbol{w}^H\boldsymbol{x} \quad (2)$$

where $\boldsymbol{w}=[\mathrm{w}_1,\mathrm{w}_2,\cdots\mathrm{w}_M]^{\mathrm{T}}$ is the weight vector and the optimal one can be calculated by solving the following optimization problem [19]

$$\min_{W}\ \boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{R}_{i+n}\boldsymbol{w} \qquad s.t.\ \boldsymbol{w}^H\boldsymbol{a}(\theta_0)=1 \quad (3)$$

where $\boldsymbol{a}(\theta_0)$ is the direction of the mainlobe and $\boldsymbol{R}_{i+n}$ is the IPN covariance matrix which can be represented as

$$\begin{aligned}\boldsymbol{R}_{i+n}&=\left(\sum_{i=1}^{D}\boldsymbol{a}(\theta_i)s_i(k)+\boldsymbol{n}(k)\right)^H\left(\sum_{i=1}^{D}\boldsymbol{a}(\theta_i)s_i(k)+\boldsymbol{n}(k)\right)\\&=\sum_{j=1}^{D}\lambda_j^2\boldsymbol{e}_j\boldsymbol{e}_j{}^H+\sigma_n^2\boldsymbol{I}\end{aligned} \quad (4)$$

where λ_j and $\boldsymbol{e}_j$ donate the jth eigenvalue and eigenvector. $\boldsymbol{I}$ donates the $N\times N$ dimensions identity matrix. The solution is

$$\boldsymbol{w}=\boldsymbol{R}_{i+n}{}^{-1}\boldsymbol{a}(\theta_0)(\boldsymbol{a}(\theta_0)^H\boldsymbol{R}_{i+n}{}^{-1}\boldsymbol{a}(\theta_0))^{-1} \quad (5)$$

which is the MVDR (minimum variance distortionless response) beamformer and this is a kind of effective side lobe interference suppression method.

Here we can see in (5) that nulls will be formed in the coming angle of the interference. However when there is a mainlobe interference in $\boldsymbol{R}_{i+n}$, the mainbeam will form a null which leads to the mainlobe distortion.

2.2 Previous work

To solve the problem mentioned above, BMP, EMP and EPCM algorithms have been proposed in recent years. For EMP, a block matrix is constructed using the eigenvector of mainlobe interference $\boldsymbol{e}_1$ which can be represented as

$$\boldsymbol{E} = \boldsymbol{I} - \boldsymbol{e}_1(\boldsymbol{e}_1{}^H\boldsymbol{e}_1)^{-1}\boldsymbol{e}_1{}^H \quad (6)$$

where $\boldsymbol{e}_1$ is estimated by the equation

$$\left|\boldsymbol{a}(\theta_0)\boldsymbol{e}_1\right|^2 \geq p\left|\boldsymbol{a}(\theta_0)\right|^2 \quad (7)$$

where p is an appropriate scalar factor which is difficult to determine in practical application.

Then processing the received data with $\boldsymbol{E}$

$$\boldsymbol{Y}=\boldsymbol{EX} \quad (8)$$

And we get the new covariance matrix

$$\boldsymbol{R}_{EMP}=\mathrm{E}\left[\boldsymbol{YY}^H\right] (9)$$

Here we can find in [11] that the mainlobe peak offset is always occur for the sake of E. For its improved one (EPCM), the direction of mainlobe $\boldsymbol{a}(\theta_0)$ is used to determine the eigenvector of mainlobe interference $\boldsymbol{e}_1$ by calculating the correlation coefficient of $\boldsymbol{a}(\theta_0)$ and interference eigenvectors $\boldsymbol{e}_i$ $i=1,2,\cdots,D$, like the following equation

$$\rho_i(\boldsymbol{a}(\theta_0),\boldsymbol{e}_{\mathrm{i}}) = \frac{\boldsymbol{a}(\theta_0)^H\boldsymbol{e}_{\mathrm{i}}}{\|\boldsymbol{a}(\theta_0)\|\|\boldsymbol{e}_{\mathrm{i}}\|} \quad i=1,2,\cdots,D \quad (10)$$

where $\rho_i(\boldsymbol{a}(\theta_0),\boldsymbol{e}_{\mathrm{i}})$ is the correlation coefficient and the eigenvector according to the maximum ρ is the mainlobe interference eigenvector and at the same time the corresponding eigenvalue can also be determined. To eliminate the mainlobe interference, the method in[12] replace the mainlobe interference eigenvalue with the estimated noise power and then reconstruct the covariance matrix which is finally used to calculate the weight vector.

The procedure to cancel the mainlobe interference of BMP is similar to the EMP method except the constructed method of the block matrix $\boldsymbol{B}$. For BMP, the information of the mainlobe interference direction θ_1 is required accurately estimated and the block matrix $\boldsymbol{B}$ is constructed based on the array structure. However when the estimated angle θ_1 is not exactly correct, the mainlobe interference can not be totally eliminate and the performance of the mainlobe array gain will be descend. Furthermore the effective DOFs are decreased too.

3. Problem Background

3.1 Covariance Matrix Reconstruction

In this paper, a method based on covariance matrix reconstruction is proposed, whose plot is shown in Figure 1. First, the direction of Arrival (DOA) of the signal is estimated. Then, the covariance is reconstructed. On this basis, signal reconstruction and interference suppression are realized.

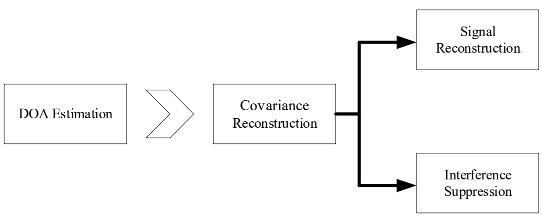


Figure 1 Diagram of the proposed method

The main idea of the proposed method is to get the constructed covariance matrix by using the Capon spectrum estimator integral over the angular sector, like methods in [15] and [16], we can use

$$P(\theta)=\frac{1}{\boldsymbol{a}(\theta)^{H}\boldsymbol{R}^{-1}\boldsymbol{a}(\theta)} \quad (11)$$

as an estimation of the spatial power spectrum. Where $\boldsymbol{a}(\theta)$ is the actual SV associate with a hypothetical direction θ based on the precise array structure information. Since the actual SV is hard to know, we use the nominal $\hat{\boldsymbol{a}}(\theta)$ instead which based on the known array structure. For special signal directions $\theta_i, i=1,2,\cdots D$, the ith SV corresponding to the ith signal can be written as $\hat{\boldsymbol{a}}(\theta_i)$. The covariance matrix without the mainlobe interference $\boldsymbol{R}_{i+n}$ can be reconstructed as

$$\boldsymbol{R}_{i+n}=\int_{\hat{\Theta}}\frac{\hat{\boldsymbol{a}}(\theta)\hat{\boldsymbol{a}}(\theta)^{H}}{\hat{\boldsymbol{a}}(\theta)^{H}\boldsymbol{R}^{-1}\hat{\boldsymbol{a}}(\theta)}d\theta \quad (12)$$

where $\hat{\Theta}$ is the complement sector of Θ which is the angular sector of the mainlobe interference.

As it is known that the spatial power spectrum is mainly distributed around the actual SV while is very small in other directions and on the other hand, directions of interferences may be widely apart. Utilizing this feature, we attempt to narrow the integral region by distributing many angular sectors for each interference according to the estimated angles $\theta_i, i=1,2,\cdots D$. For example, the mainlobe interference is located in the angular sector Θ_1, while other interferences in $\Theta_i,\ i=2,3,\cdots,D$,which are much smaller than Θ and $\hat{\Theta}$.

Here we consider an uncertainty set

$$r(\theta_i)=\left\{\hat{\boldsymbol{a}}(\theta)\,|\,\left\|\hat{\boldsymbol{a}}(\theta)-\hat{\boldsymbol{a}}(\theta_i)\right\|^2\le\varepsilon,\theta\in\Theta_i\right\},\ i=1,2,\cdots,D \quad (13)$$

where θ_i is the estimated angle of the interference by using some kind of direction finding methods. ε is a constant to make sure the uncertainty set contains all SVs of interferences, the constant ε should satisfy

$$\varepsilon\ge\max_{i=1,2\cdots D}\ \max_{\hat{a}(\theta)\in r(\theta_i)}\left\|\hat{\boldsymbol{a}}(\theta)-\boldsymbol{a}(\theta_i)\right\|^2 \quad (14)$$

Noticing that the actual SVs of interferences must be located in the set $r(\theta_i),\ i=1,2,\cdots D$, we can get each interference covariance matrix by using (11), the covariance matrix of each interference can be gotten by

$$R_i=\int_{\hat{a}(\theta)\in r(\theta_i)}\frac{\hat{\boldsymbol{a}}(\theta)\hat{\boldsymbol{a}}(\theta)^{H}}{\hat{\boldsymbol{a}}(\theta)^{H}\boldsymbol{R}^{-1}\hat{\boldsymbol{a}}(\theta)}d\theta \quad (15)$$

Here we would like to point out that in case of looking direction mismatch, the result of the estimate covariance matrix will be still accurate for the SV set $\boldsymbol{r}(\theta_i), i=1,2,\cdots,D$ constrained by the constant ε is used. The function of the parameter ε is to control the interval of the direction of the desired signals. The mainlobe interference and other interferences are separated by the angular sector $\Theta_i, i=1,2,\cdots,D$, in this point we add the interference covariance matrix up except the mainlobe interference covariance matrix to form a new covariance matrix

$$\boldsymbol{R}_I=\sum_{i=2}^{D}\boldsymbol{R}_i \quad (16)$$

On the other hand the noise power can be proximately estimated by the minimum eigenvalue of the sample matrix $\boldsymbol{R}$, which can be expressed as $\hat{\sigma}_n^2$. Then we get the reconstructed matrix

$$\hat{\boldsymbol{R}} = \boldsymbol{R}_I + \hat{\sigma}_n^2 \quad (17)$$

It is easy to see the mainlobe interference can be removed by the proposed algorithm. Based on the reconstructed covariance matrix $\hat{\boldsymbol{R}}$, the adaptive weight vector of the proposed algorithm is calculated by

$$\boldsymbol{w} = \mu \hat{\boldsymbol{R}}^{-1} \boldsymbol{a}(\theta_0) \quad (18)$$

where $\mu = 1 / \boldsymbol{a}(\theta_0)^H \hat{\boldsymbol{R}}^{-1} \boldsymbol{a}(\theta_0)$ is a scalar. We can see that the mainlobe interference contributes nothing to the calculation of the adaptive weight vector, so that there will be no null in the mainlobe.

Furthermore, the adaptive beam pattern[20] can be expressed as

$$\boldsymbol{G}_a(\theta) = \boldsymbol{G}_q(\theta) - \sum_{i=1}^{D} \frac{\lambda_i - \lambda_M}{\lambda_i} (\hat{\boldsymbol{a}}(\theta_0)^H \boldsymbol{e}_i) \boldsymbol{G}_i(\theta) \quad (19)$$

where λ_M is the smallest eigenvalue, $\boldsymbol{G}_a(\theta)$ is the adaptive beam pattern, $G_q(\theta) = \hat{\boldsymbol{a}}(\theta_0)^H \hat{\boldsymbol{a}}(\theta)$ is the quiescent beam pattern, $G_i(\theta) = \boldsymbol{e}_i^H \hat{\boldsymbol{a}}(\theta)$ is the eigen-beam which usually appears as the beam points in the direction of the interference and the maximum gain is achieved in the condition that the SV $\hat{\boldsymbol{a}}(\theta)$ is equal to or close to the SV of the *i*-th interference $\hat{\boldsymbol{a}}(\theta_i)$. As it is known that the eigenvector of the covariance matrix represents a specific direction and the corresponding eigenvalue represents the scaling factor. The maximum gain of the *i*-th eigenbeam is achieved when $G_i(\theta) = \hat{\boldsymbol{a}}(\theta_i)^H \hat{\boldsymbol{a}}(\theta_i)$ which means $\boldsymbol{e}_i$ is approximately equal to $\hat{\boldsymbol{a}}(\theta_i)$. Considering an uncertainly set $\|\boldsymbol{e}_i - \hat{\boldsymbol{a}}(\theta_i)\| \le \delta$, where δ is a positive number. The optimal solution is $\boldsymbol{e} = \hat{\boldsymbol{a}}(\theta_i)$ which can be gotten in ideal circumstances.

For EMP, the block matrix $\boldsymbol{E}$ is constructed by the eigenvector of the mainlobe interference $\boldsymbol{e}_1$ which denotes the SV of the mainlobe interference, however unfortunately it is hard to pick out $\boldsymbol{e}_1$ from all eigenvectors by using the scalar factor p in (7). However by using the proposed method in this letter, we can easily get the SV of the mianlobe interference for

$$\begin{aligned} \boldsymbol{R}_1 &= \int_{\hat{\boldsymbol{a}}(\theta) \in r(\theta_1)} \frac{\hat{\boldsymbol{a}}(\theta) \hat{\boldsymbol{a}}(\theta)^H}{\hat{\boldsymbol{a}}(\theta)^H \boldsymbol{R}^{-1} \hat{\boldsymbol{a}}(\theta)} d\theta \\ &= P_1 \hat{\boldsymbol{a}}(\theta_1) \hat{\boldsymbol{a}}(\theta_1)^H = \lambda_1 \boldsymbol{e}_1 \boldsymbol{e}_1^H \end{aligned} \quad (20)$$

where P_1 is the power of the mainlobe interference and here we can see from

$$\begin{aligned} \boldsymbol{R}_1 \hat{\boldsymbol{a}}(\theta_1) &= P_1 \hat{\boldsymbol{a}}(\theta_1) \hat{\boldsymbol{a}}(\theta_1)^H \hat{\boldsymbol{a}}(\theta_1) \\ &= P_1 \hat{\boldsymbol{a}}(\theta_1) \end{aligned} \quad (21)$$

that the actual SV of the mainlobe interference is the eigenvector of $\boldsymbol{R}_1$, even when there is noise for

$$\begin{aligned} \left(\boldsymbol{R}_1 + \sigma_n^2 \boldsymbol{I}\right) \hat{\boldsymbol{a}}(\theta_1) &= \left(P_1 \hat{\boldsymbol{a}}(\theta_1) \hat{\boldsymbol{a}}(\theta_1)^H + \sigma_n^2 \boldsymbol{I}\right) \hat{\boldsymbol{a}}(\theta_1) \\ &= \left(P_1 + \sigma_n^2 \boldsymbol{I}\right) \hat{\boldsymbol{a}}(\theta_1) \end{aligned} \quad (22)$$

we can take the eigenvector of the mainlobe interference by conducting the eigenvalue decomposition to the reconstructed mainlobe interference covariance matrix into the calculation of the adaptive weight vector in order to overcome the disadvantage mentioned in formulation(7), and then get the new block matrix $\hat{\boldsymbol{E}}$ in EMP.

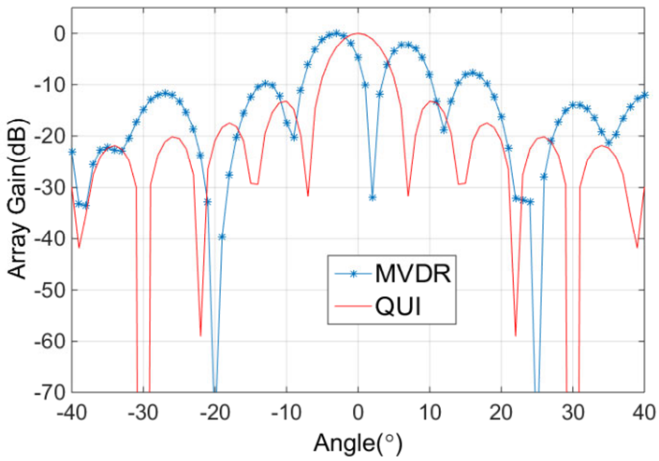
From this point of view, the proposed algorithm based on the covariance matrix reconstruction is immune to the mainlobe interference. Furthermore, it is easier to determine the mainlobe interference eigenvector to improve the algorithm in [11]. On the other hand, the proposed algorithm do not need any prior information and effective DOFs are not decreased. The proposed algorithm provides additional beamforming gain at the expense of increased computational complexity for the integral and decomposition procedure.

4. Simulation Results

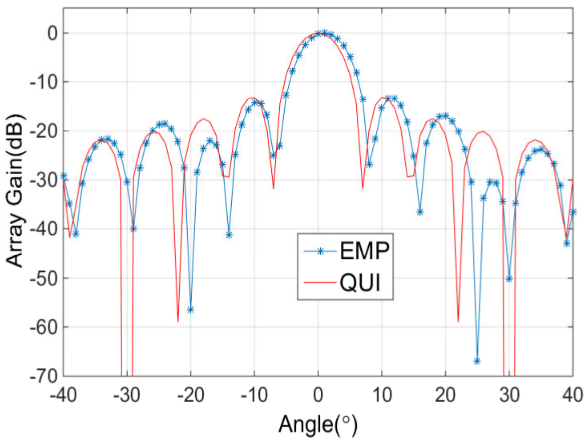
In the following simulation, an uniform linear array(ULA) with $M=10$ omnidirectional sensors spaced a half wavelength is used, Additive noise is modeled as independent complex Gaussian noise with zero mean and unit variance. The direction of the mainlobe is $\theta_0=0°$, three interferences including a mainlobe interference come from $\theta_1=-3°$ with INR 10dB and the other two $\theta_1=-20°$ and $\theta_1=25°$ with INR 40dB and 45dB. The adaptive beam pattern is obtained by conducting one Monte Carlo simulation, and the output signal-to-interference-plus-noise ratio (SINR) is calculated from an average of 200 independent Monte Carlo simulations.

Beam patterns of all mentioned algorithms including MVDR, BMP, EMP, EPCM and the proposed algorithm are compared with the quiescent (QUI) one is showed. For the proposed algorithm, we chose $\varepsilon=3°$ in order to make sure there is only one signal in each direction range.

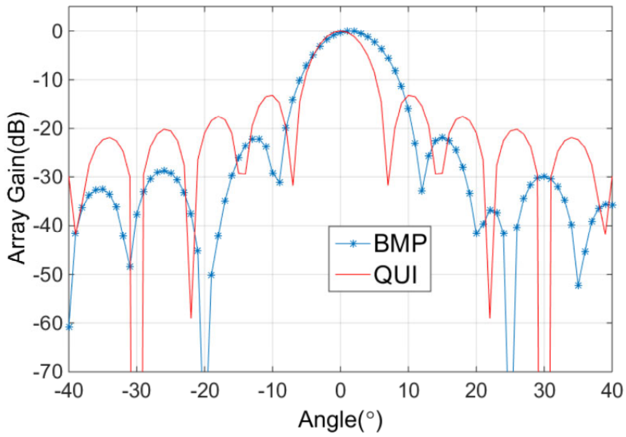
From Figure 2(e), it is found that the proposed algorithm could effectively solve the mainlobe distortion and adaptively eliminate the sidelobe interference and the beam pattern is the closest to the quiescent one, in addition, in this case, there are no interferences in the received signals and the array gain in the desired direction is also not declined. The BMP and the EMP algorithm suffer from mainlobe offset and the EMP algorithm can not form deep nulls to eliminate the sidelobe interferences. The EPCM algorithm can form a good mainlobe while the null formed to eliminate the sidelobe interference is not very deep either.



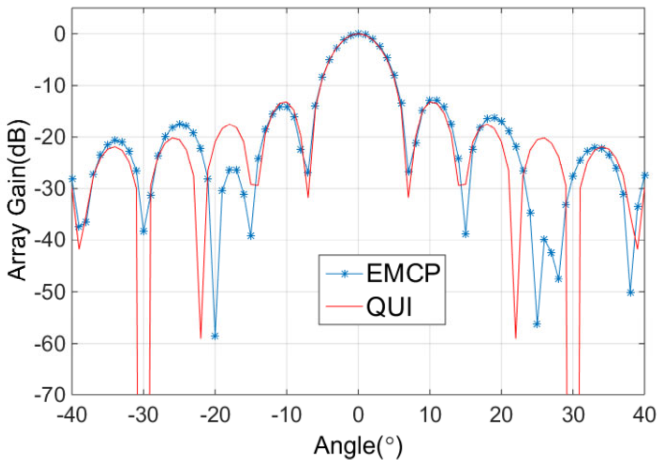
(a) MVDR



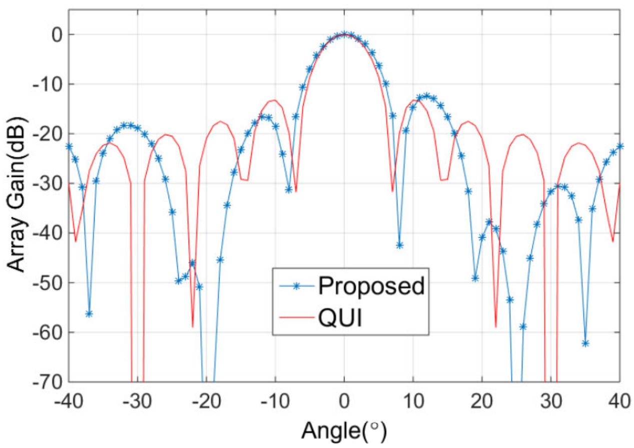
(b) EMP



(c) BMP



(d) EMCP



(e) Proposed

Figure 2 Adaptive array pattern comparison

Here we would like to compare the output SINR of the aforementioned methods versus the snapshot. The input signal to noise ratio in each sensor is 5dB. It can be seen from Figure 3 that the SINR of the proposed algorithm is the closest to the optimal SINR and the convergence is faster than EPCM and MVDR method.

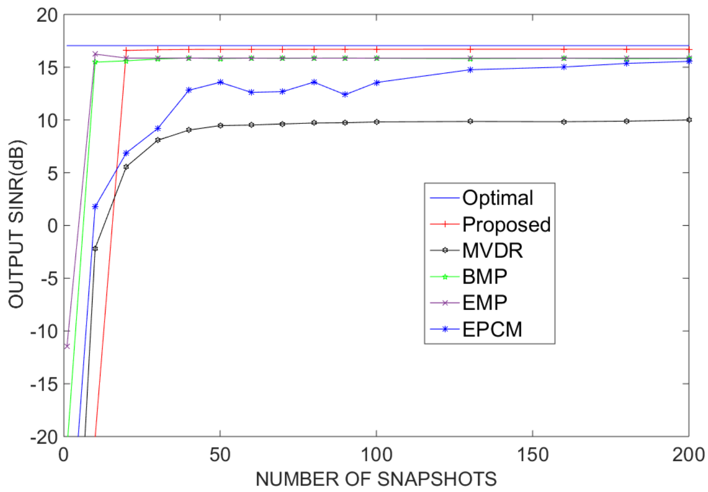


Figure 3 Output SINRS comparison

Next we show the performance of the proposed algorithm versus the mainlobe interference with different powers and compare with EMP, BMP, MVDR, EPCM method. From Figure 4, it is obvious that the proposed algorithm is robust to the mainlobe interference with different powers and keeping the closest output SINR to the optimal one.

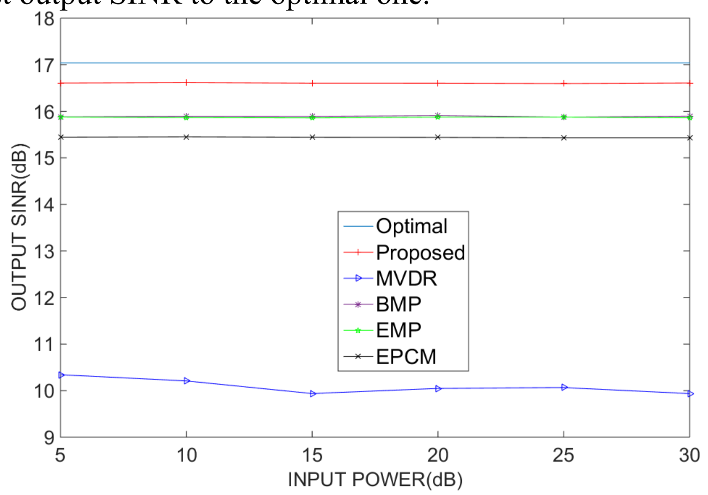


Figure 4 Output SINR comparison versus mainlobe interference with different powers

5. Conclusion

In this paper, a novel algorithm based on covariance reconstruction is proposed to suppress the mainlobe interference. In the proposed method, the direction finding method is used to estimate interference directions in the first place. And then covariance matrixes are estimated by using the spatial spectrum integral over all possible discrete angular sectors. Therefore, it is easy to get the reconstructed covariance matrix without the mainlobe interference. The proposed method does not need any prior information and the procedure is simple, which can also apply to most communication systems. Finallt, the simulation results have demonstrated that the proposed method can effectively suppress all interferences including the mainlobe interference and sidelobe interference, and can achieve an ideal output SINR.

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